ELE202: BlackBox Tutorial

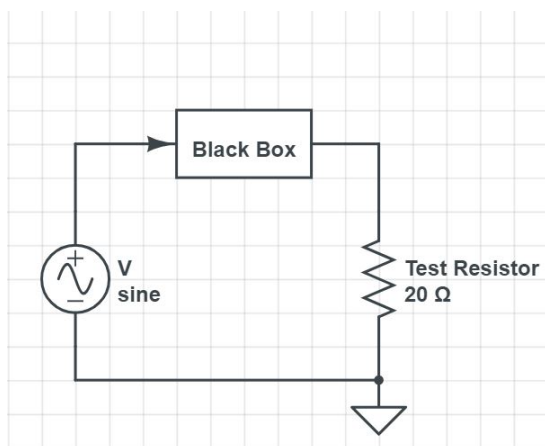Nabila Abraham
credit to Vincent Luong

Introduction

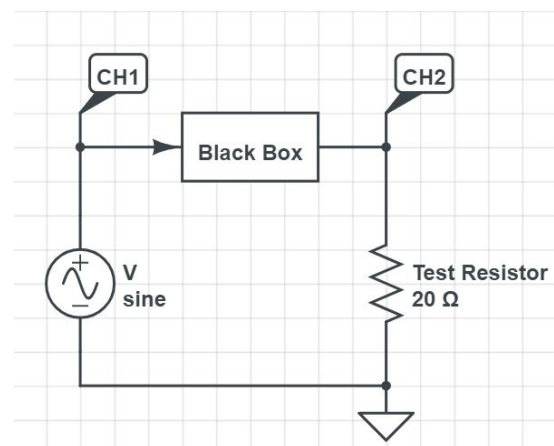There are four possible combinations in the black box:

- RC Series

- RC Parallel

- RL Series

- RL Parallel

We will use AC analysis to figure out which of the four combinations is inside the black box and also, what the values of the components are.

Step 1: Build the circuit

(a) Set up circuit in series with a test resistor of your choice. I've chosen 20Ω.

(b) Add in the oscilloscope probes.

Figure 1: Build the circuit.

Step 2: Determine RC or RL

By setting up the circuit as depicted in Figure 1b, CH1 on the oscilloscope corresponds to the voltage signal V_1 across the black box and CH2 corresponds to the voltage signal V_2 across the test resistor. However, we require a 'current' signal to do AC analysis and so we denote CH2 to be the current signal of the black box by dividing V_2 by the value of the resistor.

Therefore, CH1 $= V_1 = V_{bb}$ and CH2 $= \frac{V_2}{TestResistor} = I_{bb}$.

To determine if the circuit in the black box is an RC or RL, we observe which of the signals V_{bb} or I_{bb} are leading.

Case 1

If the oscilloscope output resembles Figure 2, then the current signal leads the voltage signal and therefore, we have a capacitive circuit (RC).
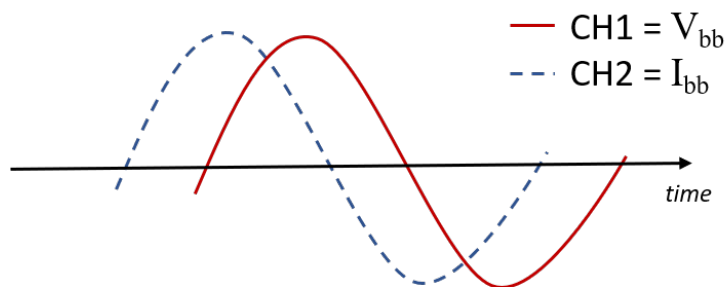

Figure 2: The current signal is *leading* and therefore, the circuit is capacitive.

Case 2

If the oscilloscope output resembles Figure 3, then the voltage signal leads the current signal and therefore, we have an inductive circuit (RL).
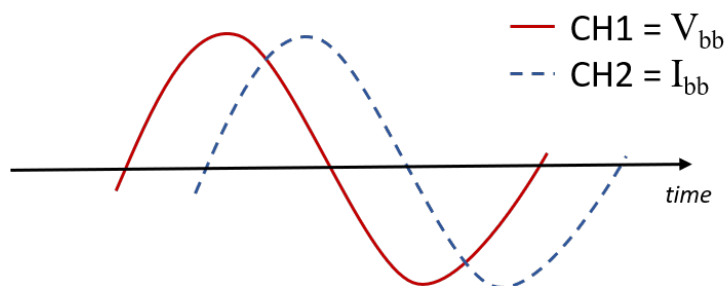

Figure 3: The voltage signal is *leading* and therefore, the circuit is inductive.

Step 3: Determine Series or Parallel

Once we know if the blackbox configuration is RC or RL, we have reduced the four choices to only two. To figure out if the circuit inside the black box is series or parallel, we can simply increase the frequency of the function generator's AC signal and observe the behavior of the phase difference between the V_{bb} and I_{bb} signals.

Case 1: Phase Angle Decreases

If on increasing the source frequency, the phase angle decreases, the circuit must be either an RC series or an RL parallel.

Why RC Series?

When we increase the frequency, the impedance of the capacitor Z_C is affected while the impedance of the resistor Z_R stays the same. As we keep increasing the frequency f, we observe that the capacitor's impedance effectively becomes a short circuit denoted by zero impedance in Equation 1. Since Z_C approaches a short circuit, the black box impedance becomes *purely resistive*, as expressed in Equation 2 and therefore the voltage signal V_{bb} will be in phase with the current signal I_{bb}.

$$\lim_{f \to \infty} Z_C = \lim_{f \to \infty} \frac{1}{2\pi f C} = \frac{1}{\infty} = 0 \tag{1}$$

(Recall, purely resistive circuits do not have any phase difference between their signals). Therefore, increasing the source frequency will reduce the phase angle between the voltage and current signals.

$$Z_{total} = Z_R + Z_C = Z_R + \frac{1}{\infty} = Z_R \tag{2}$$

Why RL Parallel?

When we increase the frequency, the impedance of the inductor Z_L is affected while the impedance of the resistor Z_R stays the same. As we keep increasing the frequency f, we observe that the inductor's impedance effectively becomes a open circuit denoted by infinite impedance in Equation 3. However, in a parallel circuit, the effective impedance is denoted by $\frac{1}{Z_{total}}$. Therefore, as expressed in Equation 4, the black box impedance becomes *purely resistive* and therefore the voltage signal V_{bb} will be in phase with the current signal I_{bb}.

$$\lim_{f \to \infty} Z_L = \lim_{f \to \infty} 2\pi f L = \infty \tag{3}$$

$$\frac{1}{Z_{total}} = \frac{1}{Z_R} + \frac{1}{Z_L} = \frac{1}{Z_R} + \frac{1}{\infty} = \frac{1}{Z_R} \tag{4}$$

(Recall, purely resistive circuits do not contain any phase difference in their signals). Therefore, increasing the source frequency will reduce the phase angle between the voltage and current signals.

Case 2: Phase Angle Increases

If on increasing the source frequency, the phase angle increases, the circuit must be either an RC parallel or an RL series.

Why RC Parallel?

When we increase the frequency, the impedance of the capacitor Z_C is affected while the impedance of the resistor Z_R stays the same. As we keep increasing the frequency f, we observe that the capacitor's impedance effectively becomes a short circuit denoted by zero impedance in Equation 5. However, in a parallel circuit, the effective impedance is denoted by $\frac{1}{Z_{total}}$. As expressed in Equation 6, the total impedance becomes *purely capacitive* and therefore the voltage signal V_{bb} lags the current signal I_{bb} by a phase difference of at most, 90 degrees.

$$\lim_{f \to \infty} Z_C = \lim_{f \to \infty} \frac{1}{2\pi f C} = \frac{1}{\infty} = 0 \tag{5}$$

$$\frac{1}{Z_{total}} = \frac{1}{Z_R} + \frac{1}{Z_C} = \frac{1}{Z_R} + \frac{1}{\frac{1}{\infty}} = \infty \tag{6}$$

(Recall, in a purely capacitive circuit, the voltage lags the current and so the signals are out of phase). Therefore, increasing the source frequency will increase the phase angle between the voltage and current signals.

Why RL Series?

When we increase the source frequency, the impedance of the inductor Z_L is affected while the impedance of the resistor Z_R stays the same. As we keep increasing the frequency f, we observe that the inductor's impedance effectively becomes a open circuit denoted by infinite impedance in Equation 7. Therefore, as expressed in Equation 8, the black box impedance becomes *purely inductive* and therefore the voltage signal V_{bb} leads the current signal I_{bb} by a phase difference of at most, 90 degrees.

$$\lim_{f \to \infty} Z_L = \lim_{f \to \infty} 2\pi f L = \infty \tag{7}$$

$$Z_{total} = Z_R + Z_L = Z_R + \infty = \infty \tag{8}$$

(Recall, in a purely inductive circuit, the voltage leads the current and so the signals are out of phase). Therefore, increasing the source frequency will increase the phase angle between the voltage and current signals.

Step 4: Solve component values

At this point, we know the configuration of the circuit inside the black box. The hardest part is done and we can simply use a bunch of Ohm's Law to figure out the R and C/L component values.

Compute Phase Angle

The black box has either a capacitor or an inductor and therefore we *know* that V_{bb} and I_{bb} will be out of phase by an angle ϕ. Therefore, we must first measure the phase angle using either the XY or YT modes discussed in previous labs. Then, depending on if the circuit is RC or RL, the phase angle will be positive or negative.

In an RC circuit, the voltage *lags* the current and so the phase angle is *negative*. In an RL circuit, the voltage *leads* the current and so the phase angle is *positive*.

Compute Total Impedance

Case 1: Series

The total circuit impedance can be described by $V = IZ$ using phasor notation:

$$Z = \frac{V < \phi}{I < \phi} \tag{9}$$

where, $V = V_{bb}$ = peak value of CH1 and $I = I_{bb} = \frac{peak\,value\,of\,CH2}{Test\,Resistor}$ and ϕ = positive or negative phase angle.

If we have figured out the components are in series, the impedance can be converted into Cartesian format to depict the real and imaginary components as below::

$$Z = RealImpedance + jImaginaryImpedance \tag{10}$$

A resistor's impedance does not contain any imaginary components so the resistance value is the real component from Equation 10. The calculation is depicted below:

$$R = RealImpedance \tag{11}$$

A capacitor/inductor's impedance contains imaginary components so we can evaluate the impedance according to Equation 12 or Equation 13, for capacitors or inductors, respectively.

Solve for C:
$$Z_C = jImaginaryImpedance = \frac{-j}{2\pi * f * C} \tag{12}$$

Solve for L:
$$Z_L = jImaginaryImpedance = j2\pi * f * L \tag{13}$$

Case 2: Parallel

The total circuit impedance can be described by $V = IZ$ using phasor notation. However, because the configuration is in parallel, we solve for $\frac{1}{Z}$ which is called *admittance Y* instead:

$$\frac{1}{Z} = \frac{I < \phi}{V < \phi}$$
$$Y = \frac{I < \phi}{V < \phi} \tag{14}$$

where, $V = V_{bb}$ = peak value of CH1 and $I = I_{bb} = \frac{peak\,value\,of\,CH2}{Test\,Resistor}$ and ϕ = positive or negative phase angle.

If we have figured out the components are in parallel, the impedance can be converted into Cartesian format to depict the real and imaginary components as in Equation 15:

$$Y = Real\,Admittance + jImaginary\,Admittance \tag{15}$$

A resistor's impedance does not contain any imaginary components so the resistance value is the real component from Equation 15. The calculation is depicted below:

$$R = \frac{1}{Real\,Admittance} \tag{16}$$

A capacitor/inductor's impedance contains imaginary components so we can evaluate the impedance according to Equation 17 or Equation 18, for capacitors or inductors, respectively.

Solve for C:

$$Y_C = jImaginary\,Admittance = \frac{1}{\frac{-j}{2\pi * f * C}} = j2\pi * f * C \tag{17}$$

Solve for L:

$$Y_L = jImaginary\,Admittance = \frac{1}{j2\pi * f * L} = -j2\pi * f * L \tag{18}$$

Sidebar

If you are confused as to how Equation 17 and Equation 18 have been reduced, recall, $j^2 = -1$ and observe the following trick:

$$\frac{1}{j} = \frac{1 * j}{j * j} = \frac{j}{-1} = -j \tag{19}$$

We prefer to have j on the numerator because then it can cancel nicely with the j in $jImaginary\,Impedance$ or $jImaginary\,Admittance$.

If any mistakes are found, please email me at **nabila.abraham@ryerson.ca**.